Solutions to Mock JEE MAIN – 4 |JEE - 2024

Physics

SINGLE CHOICE

1.(B) $V = \frac{\text{Energy}}{\text{Charge}}$

$\text{Charge} = Q = IT$

$[V] = [A][x^2] \quad \Rightarrow \quad [ML^2A^{-1}T^{-3}] = [A][L^2] \quad \Rightarrow \quad [A] = [MA^{-1}T^{-3}]$

2.(D) (A) Longest wavelength of Lyman series

$n = 2 \rightarrow n = 1$

$\therefore \quad \frac{hc}{\lambda} = E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}E_0 \qquad \ldots \text{(i)}$

(B) Shortest wavelength of Balmer series

$n = \infty \rightarrow n = 2$

$\therefore \quad \frac{hc}{\lambda'} = E_0\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{E_0}{4} \qquad \ldots \text{(ii)}$

$\therefore \quad \frac{\lambda'}{\lambda} = \frac{3}{4} \times \frac{4}{1} = 3 \quad \Rightarrow \quad \lambda' = 3\lambda$

3.(A) $\Delta T = \frac{1}{2}\alpha T \Delta\theta$

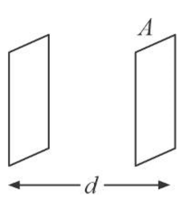
Change in time period will be

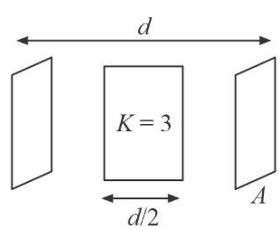
$\Delta T = \frac{1}{2}\alpha\Delta\theta T = \frac{1}{2} \times 10^{-6} \times 10 \times 10^6 = 5s$

4.(A) $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\frac{T}{29} = \left(\frac{15R}{60R}\right)^{3/2} = \frac{1}{8} \quad \Rightarrow \quad T = \frac{29}{8} \text{ days}$

5.(B)





d $C_0 = \frac{A\varepsilon_0}{d}$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$= \frac{d/2}{A\varepsilon_0} + \frac{d/2}{3A\varepsilon_0}$

$= \frac{2}{3}\frac{d}{A\varepsilon_0}$

$\therefore \quad C = \frac{3}{2} = 1.5C_0$

6.(C) $x = \frac{1}{2}at^2$

$x' = \frac{1}{2}a(t+3t)^2 = 8at^2 = 16x$

$\therefore$ Distance covered in next $3t$ seconds will be: $16x - x = 15x$

7.(A) Stopping potential is same $\Rightarrow$ $f_1 = f_2 = f_3$

Photocurrent is different $\Rightarrow$ $I_3 > I_2 > I_1$

8.(B) $V_{RMS} = \sqrt{\frac{3RT}{M}}$ $\therefore$ $V_{RMS} \propto \sqrt{T}$

$\gamma = \frac{C_P}{C_V} = \frac{5}{3}$

For adiabatic process, $TV^{\gamma-1} =$ constant

$\Rightarrow$ $T \propto \frac{1}{V^{\gamma-1}} \propto \frac{1}{V^{2/3}}$

$\frac{V_{RMS}, \text{ initial}}{V_{RMS}, \text{ final}} = \left(\frac{T_1}{T_2}\right)^{1/2} = \left(\frac{V_2}{V_1}\right)^{1/3} = 27^{1/3} = 3 : 1$

9.(D) $R_1 = \frac{2u^2 \sin\theta_1 \cos\theta_1}{g}$

$R_2 = \frac{2u^2 \sin\theta_2 \cos\theta_2}{g}$

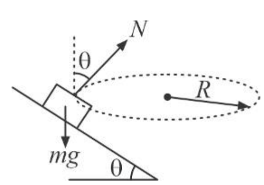
$R_1 - R_2 = \frac{2u^2}{g}(\sin\theta_1 \cos\theta_1 - \sin\theta_2 \cos\theta_2)$

$= \frac{2u^2}{g}\sin(\theta_1 - \theta_2)$

$= \frac{2u^2}{g}\sin 30° = \frac{u^2}{g}$

10.(C) $N \sin\theta = \frac{mv^2}{R}$

and $N\cos\theta = mg$



$\tan\theta = \frac{v^2}{gR}$

$\Rightarrow$ $v = \sqrt{gR\tan\theta}$

$= \sqrt{10 \times 30 \times \frac{3}{4}} = 15\ m/s$

11.(B) Induced emf in the loop,

$\varepsilon_0 = B_0 a V_0$, for $t = 0$ to $t = t_0 = \frac{a}{V_0}$

Induced current, $I_0 = \frac{B_0 a V_0}{R}$ for $t = 0$ to $t = t_0$

Work done = heat loss, $H = I_0^2 R t_0$

$= \frac{B_0^2 a^2 V_0^2}{R^2} \times R \times \frac{a}{V_0}$

$= \frac{B_0^2 a^3 V_0}{R}$

12.(C) $\vec{J} = \frac{ne^2}{m} \tau \vec{E}$

(A) $\vec{J}$ is in the same direction of $\vec{E}$
(B) $J \propto n,$ hence J will also be halved
(C) J is independent of A
(D) J depends on mass of charge carrier

13.(A) From graph $V_{\max} = 5\ m/s$ and $A = 10\ cm$

$V_{\max} = A\omega$

$\Rightarrow \quad \omega = \frac{V_{\max}}{A} = \frac{5}{10} = \frac{1}{2} s$

$T = \frac{2\pi}{\omega} = 4\pi\ s$

14.(A) $K = \frac{2\pi}{\lambda} = \pi \times 10^{-2} \quad \Rightarrow \quad \lambda = 2 \times 10^2 m$

$\omega = 2\pi \times 10^6 rad/s \quad \Rightarrow \quad f = 10^6 Hz = 1\ MHz$

Speed in medium, $v = \frac{\omega}{k} = 2 \times 10^8 m/s$

$\mu = \frac{C}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$

$B_0 = \frac{E_0}{v} = \frac{5}{2 \times 10^8} = 2.5 \times 10^{-8} T$

15.(A) Assertion is false because due to expansion, MoI will increase, and hence ω will decrease. Hence, $T > 24$ hours

16.(C) Least count, $LC = \frac{0.5\ mm}{50} = \frac{1}{100} mm$

Main scale reading $= 20 \times 0.5\ mm$
$= 10\ mm$

Circular scale reading = 35 × LC $= 35 \times \frac{1}{100} = 0.35\ mm$

Diameter = 10.35 mm

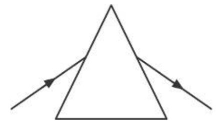
17.(D) $\Delta U =$ same in both process

$Q_{acb} - W_{acb} = Q_{adb} - W_{adb}$

$= 200 - 80 = 144 - W_{acb}$

$W_{acb} = 24J$

18.(D) Parallel beam of light will passing through prism bends towards its base.



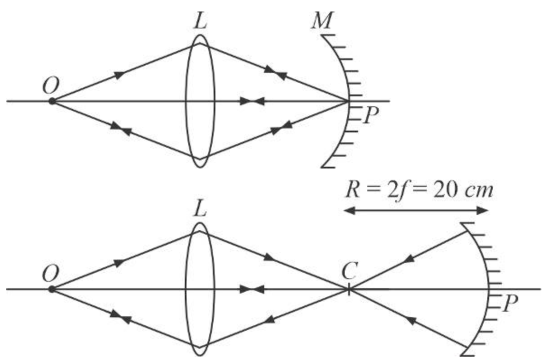
19.(D) $V = \frac{4}{3}\pi R^3$ and $2V = \frac{4}{3}\pi(R')^3 \Rightarrow 2\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R')^3 \Rightarrow R' = 2^{1/3}R$

$\Rightarrow \quad W = (2\times 4\pi[2^{1/3}R]^2\sigma) = 2^{2/3}\times 2\times 4\pi R^2\sigma = 4^{1/3}W$

20.(C) In a semiconductor diode, a layer of positive on n-side and a layer of negative charge on p-side are developed in the depletion region.

NUMERICAL TYPE

1.(20)

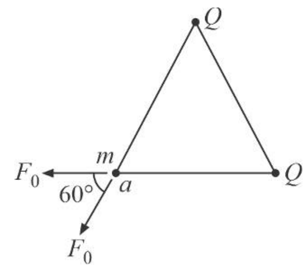


The rays should be incident either on the pole of the mirror or pass through its centre of curvature for the final image to form at the location of the object.

2.(1) $g = g_0\left(1-\frac{2h}{R}\right) = g_0\left(1-2\times\frac{32km}{6400km}\right) = g_0\left(1-\frac{1}{100}\right)$

$\therefore$ Variation in $g : \Delta g = \frac{g_0}{100}$ $\therefore$ Percentage change in g is 1%

3.(3) $F_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{l^2}$



$F_{net} = \sqrt{3}\,F_0 = ma$

$\Rightarrow \quad \sqrt{3}\times 9\times 10^9\times\frac{10^{-14}}{10^{-2}} = 3\sqrt{3}\times 10^{-3}\times a$

$\Rightarrow \quad a = 3\times 10^{9+2-14+3} = 3\ m/s^2$

4.(120) $I = 0.5\ A$

Potential difference across R_3 is 120 – 60 = 60 V

$\therefore \quad R_3 = \frac{V}{I} = \frac{60}{0.5} = 120\,\Omega$

5.(20) $U_i = \frac{Q^2}{2R}+\frac{9Q^2}{2R} = \frac{5Q^2}{R}$

When brought in contact, charges will redistribute to make potential equal

$\therefore \quad Q_1 = Q_2 = \dfrac{Q+3Q}{2} = 2Q$

$\therefore \quad U_f = \dfrac{4Q^2}{2R} \times 2 = \dfrac{4Q^2}{R}$

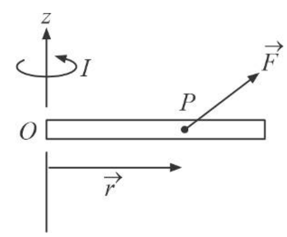
$U_{\text{loss}} = \dfrac{Q^2}{R}$

$\% \ U_{\text{loss}} = \dfrac{Q^2/R}{5Q^2/R} \times 100\% = 20\%$

6.(24) ${}_1^2H \ + \ {}_1^2H \longrightarrow {}_2^4He + Q$

$Q = 7 \times 4 - 2 \times 2 \times 1 = 24 \ MeV$

7.(10) $I = 2 \times 10^3 \, gm \cdot cm^2$



$2 \times 10^3 \times 10^{-3} \times 10^{-4} \ kg \ m^2$

$= 2 \times 10^{-4} kg \ m^2$

$\vec{r} = (2\hat{i} + 2\hat{j}) \times 10^{-2} m$

$\vec{F} = 10^{-1} \ N \, \hat{i}$

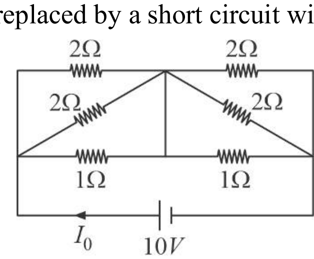
$\vec{\tau} = \vec{r} \times \vec{F} = -2 \times 10^{-3} N \cdot m \, \hat{k}$

$\vec{\tau} = I \, \vec{\alpha}$

$\Rightarrow \quad -2 \times 10^{-3} \, \hat{i} = 2 \times 10^{-4} \, \vec{\alpha}$

$\Rightarrow \quad \vec{\alpha} = 10 \, rad/s^2 \, \vec{k}$

8.(10) At steady state, inductor can be replaced by a short circuit wire.



$\text{R}_{\text{eq}} = 1 \, \Omega$

$I_0 = \dfrac{10}{1} = 10 \ A$

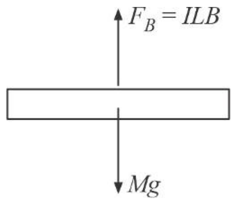
9.(9) $E = \dfrac{1}{2} KA^2$

$U = \dfrac{1}{2} Kx^2 = \dfrac{1}{2} K \left(\dfrac{A}{3}\right)^2 = \dfrac{E}{9}$

$K = E - U = \dfrac{8}{9} E$

$$\frac{K}{U} = \frac{8}{1} = \frac{a}{b}$$

10.(1)



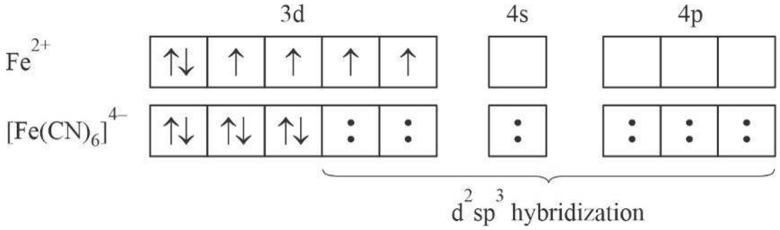
$ILB = Mg$

$$\Rightarrow \qquad I = \frac{25}{1000} \times 10 \times \frac{100}{50} \times \frac{10}{5} = 1\,A$$

Chemistry

SINGLE CHOICE

1.(B) $[Fe(CN)_6]^{4-}$ involves d^2sp^3 hybridization. As CN^- is a strong ligand, the pairing of electrons occurs, thus, it is a diamagnetic ion.



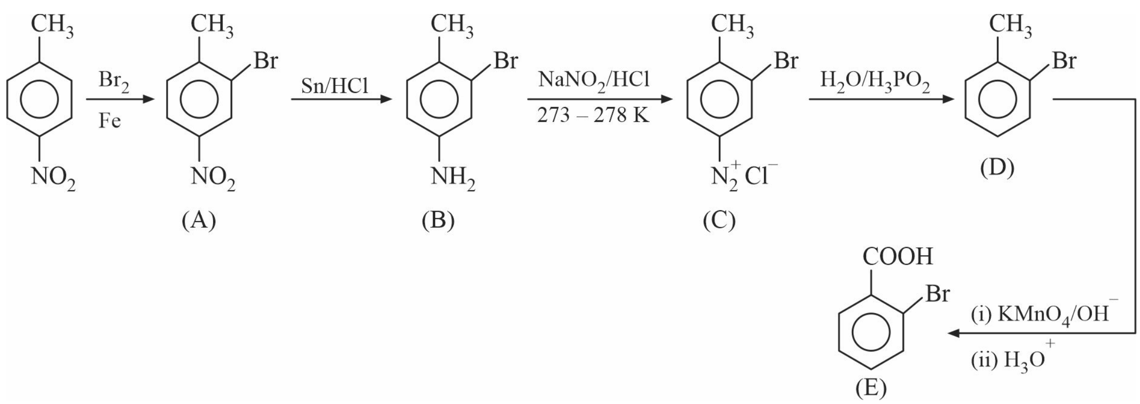
All orbitals are doubly occupied

2.(C) Theoretical

3.(C) In isoelectronic ions, the atomic size decreases as $\frac{z}{e}$ ratio increases.

$$S^{2-}\left(\frac{z}{e}\right)=\frac{16}{18}, Cl^-\left(\frac{z}{e}\right)=\frac{17}{18}, Ar\left(\frac{z}{e}\right)=\frac{18}{18}, K^+\left(\frac{z}{e}\right)=\frac{19}{18}, Ca^{2+}\left(\frac{z}{e}\right)=\frac{20}{18}$$

4.(B)



5.(C) Theoretical

6.(C) Moles of alcohol = moles of CH_4 liberated $=\frac{0.22}{M_{alcohol}}=\frac{56}{22400}$

$M_{alcohol}=88$

General formed of alcohol ; $C_nH_{2n+2}O$

$12\times n+(2n+2)\times 1+1\times 16=88$

$n=5$

Molecular formula of alcohol : $C_5H_{12}O$

7.(A) $[Ag(CN)_2]^-$

$Ag^+:4d^{10}$, 0 unpaired electrons

$[Cu(CN)_4]^{3-}$

$Cu^+:3d^{10}$, 0 unpaired electrons

$[Cu(NH_3)_4]^{2+}$

$Cu^{2+} : 3d^9$, 1 unpaired electron

$[Fe(CN)_6]^{4-}$

$Fe^{2+} : 3d^6$, CN^- is a strong field ligand hence all the electrons of 3d will be paired.

8.(A) Gram atomic mass of C = 12 gm

Gram atomic mass of Al = 27 gm

Gram atomic mass of C and Al contains Avogadro number of atoms.

9.(D) Theoretical

10.(D) **(i)** $Na_3PO_4 = \frac{164}{2} = 82$

(ii) $Ca_3(PO_4)_2 = \frac{310}{6} = 51.67$

(iii) $Na_2CO_3 = \frac{106}{1} = 106$

(iv) $H_2C_2O_4 \cdot 2H_2O = \frac{126}{2} = 63$

11.(A) $\underset{}{CH_3COOH} \xrightarrow{LiAlH_4} \underset{(X)}{CH_3CH_2OH} \xrightarrow[573\text{ K}]{Cu} \underset{(Y)}{CH_3CHO} \xrightarrow[NaOH]{Dilute} \underset{\substack{(Z)\\(\text{Aldol})}}{CH_3-\underset{\substack{|\\OH}}{CH}-CH_2-CHO}$

12.(C) $CH_3-C\equiv CH \xrightarrow{NaNH_2} CH_3-C\equiv \ddot{C}^-Na^+ \xrightarrow{CH_3I} CH_3-C\equiv C-CH_3 \xrightarrow[\text{dil. } H_2SO_4]{HgSO_4} CH_3-CH_2-\overset{\substack{O\\||}}{C}-CH_3$

13.(C) Volume of NH_4OH solution = Volume of HCl solution = V (in litres)

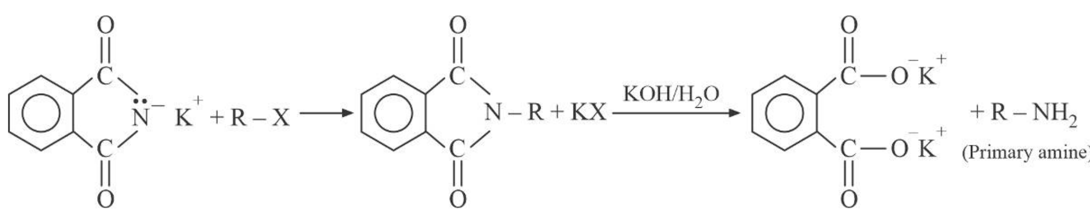
Moles of $NH_4OH = 0.1 \times V$

Moles of $HCl = 0.1 \times V$

NH_4OH	+	HCl	$\longrightarrow$	NH_4Cl	+	H_2O
$0.1 \times V$		$0.05 \times V$		0		–
$0.05 \times V$		0		$0.05 \times V$		–

Since, both NH_4OH and NH_4Cl are present hence it will behave as buffer solution.

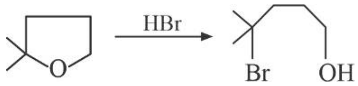
14.(D)



15.(A) Bond order $= \frac{1}{2}(N_b - N_a)$

N_b = Number of bonding electrons

N_a = Number of antibonding electrons

16.(A) 

17.(B) % ionic character $= \frac{\text{Observed dipole moment}}{\text{Theoretical dipole moment}} \times 100$

Theoretical dipole moment of a 100% ionic character

$= e \times d = (1.6 \times 10^{-19} C) \times (1.41 \times 10^{-10} m)$

$= 2.256 \times 10^{-29} C \cdot m$

18.(B) $Fe(OH)_3$ reddish brown colour

19.(C) Stability of carbocation is governed by resonance, hyper-conjugation and inductive effect.

20.(B) Al has I.P. lower than Mg and higher than Na as it has configuration $3s^2 3p^1$.

NUMERICAL TYPE

1.(1) Number of revolution $= \frac{v}{2\pi r} = \frac{v_0 z}{n \times 2\pi \frac{n^2 a_0}{z}} = N$

$\Rightarrow \quad N = \frac{v_0}{2\pi a_0}\left(\frac{z^2}{n^3}\right) = \frac{kz^2}{n^3}$

For He^+ in 2nd orbit $N_2 = \frac{k(2)^2}{2^3} = \frac{k}{2}$

$\Rightarrow \quad 2N_2 = k = N_1$ (number of revolutions in H-atom)

$\Rightarrow \quad N_1 = k = \frac{k}{n^3} \Rightarrow n = 1$

2.(3) For the reaction,

$A + D \rightleftharpoons P + \frac{C}{2}$

$K = \frac{6}{\sqrt{4}} = 3$

3.(1) The balanced half-reaction is $2H^+ + NO_2^- + e^- \longrightarrow NO + H_2O$.

4.(4) General formula of alkane : C_nH_{2n+2}

$12 \times n + (2n+2) \times 1 = 72$

$14n = 70 \Rightarrow n = 5$

Molecular formula : C_5H_{12}

Alkane is $\begin{array}{c} CH_3 \\ | \\ CH_3-C-CH_3 \\ | \\ CH_3 \end{array}$

Number of 1° hydrogen = 12

5.(4) $3.72 = (1+x)\times 1.86 \times 0.4 \Rightarrow x = 4$

6.(6) $t_{1/2}(i) = 2\text{ hr}$

$t_{1/2}(ii) = 8\text{ hr}$

7.(3) Moles of AgCl formed $= \dfrac{4.305}{143.5} = 0.03$

Moles of complex $= 0.1\times\dfrac{100}{1000} = 0.01$

Mole of AgCl formed = 3 $\times$ moles of complex

Hence $3\,Cl^-$ ions should be produced by complex

Complex is $[Co(NH_3)_6]Cl_3$

Charge on complex = +3

8.(2) $\alpha = \dfrac{\Lambda^c}{\Lambda^\infty} = \dfrac{10}{200} = \dfrac{1}{20}$

$\Rightarrow \quad [H^+] = C\alpha = 0.2\times\dfrac{1}{20} = \dfrac{1}{100} \quad \Rightarrow \quad pH = -\log\dfrac{1}{100} = 2$

9.(8) $w = -p_{ex}(V_f - V_i) = -2\times 40 = -80\text{ L bar} = -8\text{kJ}$

The negative sign shows that work is done by the gas.

10.(4) $CN^+ = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2px}^2 = \pi_{2py}^2$

Number of anti-bonding electrons = 4

Mathematics

SINGLE CHOICE

1.(C) Let $z = a + bi$

$|z|^2 = a^2 + b^2$

So, $z + |z| = 2 + 8i$

$a + bi + \sqrt{a^2 + b^2} = 2 + 8i$

$a + \sqrt{a^2 + b^2} = 2, b = 8$

$a + \sqrt{a^2 + 64} = 2$

$a^2 + 64 = (2 - a)^2 = a^2 - 4a + 4$

$4a = -60, \ a = -15$

Thus, $a^2 + b^2 = 225 + 64 = 289$ $\therefore$ $|z| = \sqrt{a^2 + b^2} = \sqrt{289} = 17$

2.(A) Equation of the line is

$$\frac{x-2}{1} = \frac{y+2}{-3} = \frac{z-5}{2} = \lambda \quad \ldots \text{(i)}$$

Hence any point on the line (i) can be taken as

$\Rightarrow$ $x = \lambda + 2$

$y = -(3\lambda + 2)$

$z = (2\lambda + 5)$

For some λ point lies on the plane

$2x - 3y + 4z = 163$ … (ii)

$2(\lambda + 2) + 3(3\lambda + 2) + 4(2\lambda + 5) = 163$

This gives, $19\lambda = 133$

$\Rightarrow$ $\lambda = 7$

Hence, $P = (9, -23, 19)$

Also equation (i) intersect YZ plane *i.e.*, $x = 0$

$\Rightarrow$ $\lambda + 2 = 0$, hence $\lambda = -2$

$\therefore$ $Q(0, 4, 1)$

So, $PQ = \sqrt{9^2 + 27^2 + 18^2} = 9\sqrt{1 + 3^2 + 2^2} = 9\sqrt{14}$

$\therefore$ $a = 9$ and $b = 14$

Hence, $a + b = 23$

3.(B) $4^{87} + 6^{87}$

$(5-1)^{87} + (5+1)^{87}$

$= (1+5)^{87} - (1-5)^{87}$

$= 2[\,^{87}C_1 \cdot 5 + {}^{87}C_3 \cdot 5^3 + \ldots + {}^{87}C_{87} \cdot 5^{87}]$

$= 10 \cdot 87 + 2[\,^{87}C_3 \cdot 5^3 + \ldots + {}^{87}C_{87} \cdot 5^{87}]$

= 870 + an expression divisible by 25

$\Rightarrow \quad \dfrac{870}{25} = 34 + \dfrac{20}{25} \qquad \Rightarrow \quad$ Remainder is 20

4.(D) $LHS = \vec{d} - \vec{a} + \vec{d} - \vec{b} + \vec{h} - \vec{c} + 3(\vec{g} - \vec{h})$

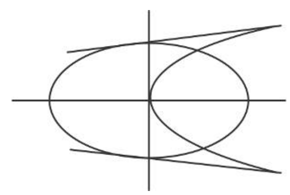
$= 2\vec{d} - (\vec{a} + \vec{b} + \vec{c}) + 3\dfrac{(\vec{a} + \vec{b} + \vec{c})}{3} - 2\vec{h}$

$= 2\vec{d} - 2\vec{h} = 2(\vec{d} - \vec{h}) = 2\overrightarrow{HD} \qquad \Rightarrow \qquad \lambda = 2$

5.(A) $y = \dfrac{x}{2} + 2$ is tangent on the ellipse then

$4 = 4 \cdot \left(\dfrac{1}{2}\right) + b^2 \qquad \Rightarrow \qquad b^2 = 3$

Parabola is, $\quad y = mx + \dfrac{1}{m}$



Using condition of tangency,

$\dfrac{1}{m^2} = 4m^2 + 3$

$4y^2 + 3y - 1 = 0 \qquad$ (when $m^2 = y$)

$4y^2 + 4y - y - 1 = 0$

$\Rightarrow \quad 4y(y+1) - (y+1) = 0$

$\Rightarrow \quad y = \dfrac{1}{4}\ ;\ y = -1$

$m = \pm\dfrac{1}{2}$

$y = \dfrac{x}{2} + 2$ or $y = -\dfrac{x}{2} - 2$

$\Rightarrow \quad 2y + x + 4 = 0 \qquad$ (other tangent)

6.(B) $\displaystyle\int \frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}\,dx$

Put $\quad x^2 + 3x = t \quad \Rightarrow \quad (2x+3)dx = dt$

$\displaystyle\int \frac{dt}{t(t+2)+1} = \int \frac{dt}{(t+1)^2} = C - \frac{1}{t+1} = C - \frac{1}{x^2+3x+1}$

$\Rightarrow \quad a = 1,\ b = 3,\ c = 1 \qquad \Rightarrow \qquad a + b + c = 5$

7.(C) Player A can win if A throws (1, 6) or (6, 1) and B throws ((1, 1,), (2, 2), (3, 3), (4, 4), (5, 5) or (6, 6)). Thus the number of ways is 12.

Similarly the number of ways in which B can win is 12.

Total number of ways in which either A wins or B wins = 24.

Thus the number of ways in which none of the two wins $= 6^4 - 24$.

$\therefore$ The required probability $= \dfrac{6^4 - 24}{6^4} = \dfrac{53}{54}$

8.(A) $\displaystyle\int \frac{1}{\sqrt[3]{x^2}\sqrt[3]{(2+3x)^4}}\,dx$

$$= \int \frac{1}{x^{2/3}(2+3x)^{4/3}}\,dx$$

Let $\dfrac{2+3x}{x} = t \Rightarrow \dfrac{-2}{x^2}\,dx = dt$

$$= -\frac{1}{2}\int \frac{1}{t^{4/3}}\,dt = \frac{3}{2}t^{-1/3} = \frac{3}{2}\left[\frac{x}{2+3x}\right]^{1/3} + c$$

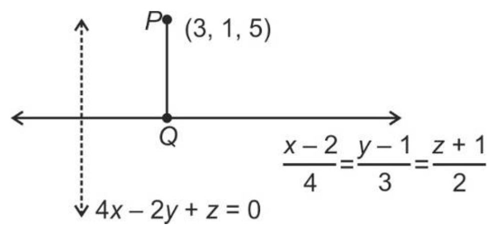
9.(A) Let coordinates of Q are $(4\lambda+2,\ 3\lambda+1,\ 2\lambda-1)$

Direction ratios of PQ are $< 4\lambda-1,\ 3\lambda,\ 2\lambda-6 >$

$\Rightarrow \quad 4(4\lambda-1) - 2(3\lambda) + 1(2\lambda-6) = 0$

$\Rightarrow \quad \lambda = \dfrac{5}{6}$

$\therefore \quad Q \equiv \left(\dfrac{16}{3}, \dfrac{7}{2}, \dfrac{2}{3}\right)$



10.(D) Consider the roots of the equation $z^p = 1$, the roots are

$$\cos\frac{2\pi k}{p} + i\sin\left(\frac{2\pi k}{p}\right) = z_k \qquad (Say)$$

$$z^{p-1} + z^{p-2} + \ldots + z^2 + z + 1 = (z-z_1)(z-z_2)\ldots(z-z_{k-1})$$

Put $z = 1$ and apply modulus

11.(D) $\dfrac{|adj\ B|}{|C|} = \dfrac{|adj(adj\ A)|}{|3A|} = \dfrac{|A|^{2^2}}{3^3|A|} = \left(\dfrac{|A|}{3}\right)^3$

$$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 1(13) - 1(-1) + 2(-4) = 6$$

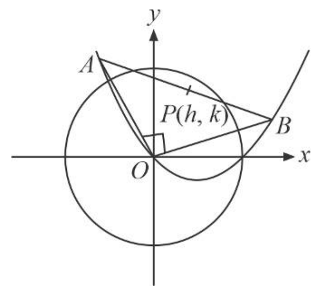
Hence, $\dfrac{|adj\ B|}{|C|} = \left(\dfrac{6}{3}\right)^3 = 8$

12.(D) Coefficient of variation $C.V = \frac{\sigma_x}{\bar{x}} \times 100$

$\Rightarrow \quad 60 = \frac{20}{\bar{x}} \times 100 \Rightarrow \quad \bar{x} = 33.33$

13.(D) Equation of chord of the circle with mid-point (h, k) is $hx + ky = h^2 + k^2$

On homogenizing the parabola, we get



$$x^2 - \frac{(x+y)(hx+ky)}{h^2+k^2} = 0$$

$\Rightarrow \quad (h^2+k^2)x^2 - (x+y)(hx+ky) = 0$

Now coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore \quad h^2 + k^2 - h - k = 0$

$\Rightarrow \quad x^2 + y^2 - x - y = 0$

$\Rightarrow \quad$ Circle with centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius $= \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$

14.(A) Toys in group 1 1 2 $\Rightarrow \frac{4!}{1!1!2!2!} \times 3! = 36$

Marbles O O ◎ ◎ $= {}^4C_2 = 6$

$\Rightarrow \quad$ Total ways $= 36 \times 6 = 216$

15.(D) When A has B or C to his right we have AB or AC.

When B has C or D to his right we have BC or BD.

Thus $\Rightarrow$ We must have ABC or ABD or AC and BD.

For $\boxed{A\,B\,C,}\ D, E, F$ on a circle number of ways = 3! = 6

For $\boxed{A\,B\,D,}\ C, E, F$ on a circle number of ways = 3! = 6

For $\boxed{A\,C}, \boxed{B\,D}\ E, F$ the number of ways = 3! = 6

Total = 18

16.(B) Here,

$$\therefore \quad g(x) = \begin{cases} 4 + 2x - x^2 & , \text{ if } 0 \le x < 1 \\ 5 & , \text{ if } 1 \le x \le 2 \\ 1 + 4x - x^2 & , \text{ if } 2 < x < 3 \\ 6 & , \text{ if } 3 \le x \le 5 \end{cases}$$

Which is not continuous at $x = 3$

17.(A) $(1 - 2x + 5x^2 + 10x^3)[C_0 + C_1x + C_2x^2 + \ldots]$

$= 1 + a_1x + a_2x^2 + \ldots$

$a_1 = n-2$ and $a_2 = \frac{n(n-1)}{2} - 2n + 5$

Put $a_1^2 = 2a_2$

$(n-2)^2 = n(n-1) - 4n + 10$

$n^2 - 4n + 4 = n^2 - 5n + 10$

$n = 6$

18.(B) Ordered pairs that should be added is (2, 2) (3, 3) (4, 4) (3, 2) (4, 3)

19.(C) $\vec{r} = (\vec{a} \times \vec{b}) \sin x + (\vec{b} \times \vec{c}) \cos y + 2(\vec{c} \times \vec{a})$

$\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \quad [\vec{a}\,\vec{b}\,\vec{c}]\,(\sin x + \cos y + 2) = 0 \quad \Rightarrow \quad [\vec{a}\,\vec{b}\,\vec{c}] \neq 0 \quad \Rightarrow \quad \sin x + \cos y = -2$

This is possible only when $\sin x = -1$ *and* $cos y = -1$

For $x^2 + y^2$ to be minimum $x = -\frac{\pi}{2}$ and $y = \pi$

$\Rightarrow$ Minimum value of $(x^2 + y^2) = \frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}$

20.(B) Let $a = x, b = x + d_1, c = x + 2d_1$ and $d = x + 3d_1$ (as *a*, *b*, *c*, *d* are in A.P.)

Hence *a*, *b*, *d* are in G.P.

$\therefore \quad (x + d_1)^2 = x(x + 3d_1)$

$2xd_1 + d_1^2 = 3xd_1$

$d_1^2 = xd_1$

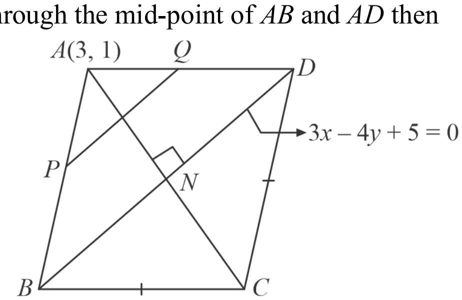
$\Rightarrow \quad x = d_1$ (as $d_1 \neq 0$)

Hence, $a = d_1,\ b = 2d_1,\ c = 3d_1,\ d = 4d_1$

$\therefore \quad \frac{ad}{bc} = \frac{d_1 \cdot 4d_1}{2d_1 \cdot 3d_1} = \frac{4}{6} = \frac{2}{3}$

NUMERICAL TYPE

1.(1) Because line *PQ* passes through the mid-point of *AB* and *AD* then



$PQ \parallel BD$ and $AN \perp BD$, so *M* is also the mid-point of *AN* (by similarity)

$AN = \left| \frac{9 - 4 + 5}{5} \right| = 2$

$AM = 1$

Suppose equation of PQ

$\Rightarrow \quad 3x - 4y + c = 0$

Perpendicular from A on PQ is

$$AM = \left|\frac{5 + C}{5}\right| = 1$$

$\Rightarrow \quad C = 0$ or -10 then equation of PQ can be $3x - 4y = 0$ or $3x - 4y - 10 = 0$

But PQ has unit distance from BD

Hence equation of PQ must be $3x - 4y = 0$

$\Rightarrow \quad ax + by + c = 0 \qquad \Rightarrow \quad |a + b + c| = |3 - 4 + 0| = 1$

2.(216) $(\sqrt{x}\log_b x)^3 = b^{56}$ . . . (1)

$(\log_b x)^{54} = x$

$$\Rightarrow \quad \left(\sqrt{x}\,x^{\frac{1}{54}}\right)^3 = b^{56} \qquad . . . (2)$$

$\Rightarrow \quad x^{\frac{14}{9}} = b^{56} \qquad \Rightarrow \quad x = b^{36}$

From (2), we get

$(36)^{54} = x = b^{36}$

$\Rightarrow \quad b = (36)^{3/2} = 216$

3.(27) All the possible numbers are ${}^9C_5$ (none containing the digit 0) = 126

Total numbers starting with $1 = {}^8C_4 = 70$

1				

(Using 2, 3, 4, 5, 6, 7, 8, 9)

Total starting with $23 = {}^6C_3 = 20$

2	3			

(Using 4, 5, 6, 7, 8, 9)

Total starting with $245 = {}^4C_2 = 6$

2	4	5		

(Using 6, 7, 8, 9)

97th number =

2	4	6	7	8

4.(2) Since, $|x| = 1$

$\therefore \quad x = \pm 1$

$$y = xe^{|x|} = \begin{cases} xe^{-x} & , \quad -1 < x < 0 \\ xe^{x} & , \quad 0 \le x < 1 \end{cases}$$

$$\therefore \quad \text{Required area} = \left|\int_{-1}^{0} xe^{-x}dx\right| + \left|\int_{0}^{1} xe^{x}dx\right|$$

$$= \left|\left\{-xe^{-x} - e^{-x}\right\}_{-1}^{0}\right| + \left|\left\{xe^{x} - e^{x}\right\}_{0}^{1}\right| = 2\ sq.\ unit$$

5.(0) P is the point of intersect of two perpendicular tangents to the parabola $y^2 = 8x,\ 4a = 8 \Rightarrow a = 2$

Hence P must lie on directrix $x + a = 0$

or $x + 2 = 0$

P also lies on tangent $y = x + 2$

$\therefore$ $x = -2$ and $y = 0$ hence point is $(-2, 0)$

6.(7) The equation of the tangent at $(5\cos\theta, 2\sin\theta)$ is $\frac{x}{5}\cos\theta + \frac{y}{2}\sin\theta = 1$

If it is a tangent to the circle then $\frac{1}{\sqrt{\frac{\cos^2\theta}{25} + \frac{\sin^2\theta}{4}}} = 4 \Rightarrow \cos\theta = \frac{10}{4\sqrt{7}}, \sin\theta = \frac{\sqrt{3}}{2\sqrt{7}}$

Let A and B the points where the tangent meets the coordinate exis then

$A\left(\frac{5}{\cos\theta}, 0\right), B\left(0, \frac{2}{\sin\theta}\right),\ L = \sqrt{\frac{25}{\cos^2\theta} + \frac{4}{\sin^2\theta}} = \frac{14}{\sqrt{3}}$

7.(4) Normal to $xy = c^2$ is $y - \frac{c}{t} = t^2(x - ct)$

Solving with $xy = -c^2$ we get

$x\left(\frac{c}{t} + t^2(x - ct)\right) + c^2 = 0 \qquad t^2x^2 + \left(\frac{c}{t} - ct^3\right)x + c^2 = 0$

For equal roots $\left(\frac{1}{t} - t^3\right)^2 - 4t^2 = 0 \Rightarrow 4$ values are possible

8.(11) Given lines are $\frac{x-5}{0} = \frac{y}{4-\alpha} = \frac{z}{-1}$ and $\frac{x-\alpha}{0} = \frac{y}{-2} = \frac{z}{2-\alpha}$ are coplanar

$$\therefore \quad \begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 4-\alpha & -1 \\ 0 & -2 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow$ $\alpha^3 - 11\alpha^2 + 36\alpha - 30 = 0$

$\Rightarrow$ Sum of all possible values of α

$= \alpha_1 + \alpha_2 + \alpha_3 = \frac{-(-11)}{1} = 11$) and it can be seen that all values of α are real

9.(90) $C : x^2 - y^2 = 1$

$E:\ focus\ (\pm\sqrt{2}, 0), e = \sqrt{2/3}$

$a = \sqrt{3}$

$\frac{x^2}{3} + y^2 = 1$

and therefore the angle is a right angle.

10.(3) For range $[0, \infty)$, $D = 0$

$$4(\sin^{-1}\beta)^2 - 4\left(\frac{\pi}{\sqrt{2}} + \sin^{-1}\alpha\right)\left(\frac{\pi}{\sqrt{2}} - \sin^{-1}\alpha\right) = 0$$

$$\therefore \qquad (\sin^{-1}\alpha)^2 + (\sin^{-1}\beta)^2 = \frac{\pi^2}{2}$$

$$\Rightarrow \qquad (\sin^{-1}\alpha)^2 + (\sin^{-1}\beta)^2 = \frac{\pi^2}{2} \qquad \Rightarrow \qquad \sin^{-1}\alpha = \pm\frac{\pi}{2}$$

$$\sin^{-1}\beta = \pm\frac{\pi}{2} \qquad \Rightarrow \qquad \alpha = \pm 1,\ \beta = \pm 1$$